Life of Fred

Decimals and Percents

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A Note to Students

This is the last arithmetic book. After you finish the 192 pages of this book you will be ready for algebra. There are 33 chapters. Each chapter is a lesson. Just like in *Life of Fred: Fractions,* after each five chapters you will come to **The Bridge**, which will give you a chance to show that you know the math before you move on to the next chapter.

The main danger in the *Life of Fred* books is that the readers enjoy them too much. The temptation is to zoom through the chapters reading about the adventures of Fred. Here is a secret:

LEARNING ABOUT DECIMALS AND PERCENTS
IS AN IMPORTANT PART OF THIS BOOK.

We continue our story of Fred where we left off at the end of *Life of Fred: Fractions.* As before, when I am writing I will use Times New Roman typeface. When Fred is thinking, he'll use this typeface. And when you, my reader, voice your questions (or complaints), ***you will use this typeface.***

Now that we have settled all of that, feel free to skip the rest of what is called the "front matter" and turn to page 13 to find out what Fred did after he opened the box that *didn't* contain his bicycle.

A Note to Teachers and Autodidacts

Mary Poppins was right: A spoonful of sugar can make life a little more pleasant. It is surprising that so few arithmetic books have figured that out.

Some arithmetic books omit the sugar—which is like lemonade without any sweetener. They give you a couple of examples followed by a zillion identical problems to do. And they call that a lesson. No wonder students aren't eager to read those books.

At the other extreme are the books that are just pure sugar—imagine a glass of lemonade with so much sugar in it that your spoon floats. The pages are filled with color and happy little pictures to show you how wonderful arithmetic is. The book comes with ① a teachers' manual, ② a computer disc, ③ a test booklet, and ④ a box of manipulatives. And they are so bŭsy entertaining the reader that they don't teach a lot of math. This second approach is also usually quite expen$ive.

We'll take the Goldilocks approach: not too sour and not too sweet. We will also include a lot of mathematics. (Check out the Contents on page 10.) How many arithmetic books include both forms of the Goldbach Conjecture? (See chapter 17.) The reader will be ready for algebra after completing this book.

This book covers one afternoon and evening of Fred's life and continues the story from *Life of Fred: Fractions.* Every piece of math first happens in his life, and then we do the math. It is all motivated by real life. When is the last time you saw prime numbers actually *used* in everyday life? They are needed in this book when the cavalry is getting ready to attack what the newspaper calls the "Death Monster."

FACTS ABOUT THE BOOK

Each chapter is a lesson. Thirty-three chapters = 33 lessons.

At the end of each chapter is a *Your Turn to Play*, which gives the student an opportunity to work with the material just presented. The

answers are all supplied. The questions are not all look-alike questions. Some of them require . . . thought! Each *Your Turn to Play* incorporates some review material. The students will get plenty of opportunity to keep using the material they have learned.

At the end of every five chapters is **The Bridge**, ten questions reviewing everything learned up to that point. If students want to get on to the next chapter, they need to show *mastery* of what has been covered so far. They need to get nine or more questions correct✶ in order to move on to the new material. If they don't succeed on the first try, there is a second set of ten questions—a second try. And a third try. And a fourth try. And a fifth try. Lots of chances to cross the bridge.

Don't let your students move on without showing mastery of the previous math. If you want to make your students pass two of the five bridges instead of just one, that is okay with me.

At the end of the book is **The Final Bridge**, consisting of twenty questions. Again, five tries are offered.

RULES OF THE GAME

For now, students should put aside their calculators. This is the last chance we have to cement in place their addition and multiplication facts (which they should have had memorized before they began *Life of Fred: Fractions.*) I balance my checkbook each month without a calculator just to keep in practice.

Once the students get to algebra they can take their calculators out of their drawers and use them all they like.

When the students are working on the *Your Turn to Play* or **The Bridge** sections, they should write out their answers. When they are working on a **Bridge**, they should complete the whole quiz before checking their answers. Many teachers and parents will want to grade the students' answers themselves in order to monitor their progress. Mastery of the material is much more important than speed.

✶ The answers to all of the Bridge questions are given right before the index in the back of this book.

FINAL THOUGHTS

These *Life of Fred* books are designed to teach the material. They are not merely repositories of examples and homework problems. It is so important that kids

learn
how to learn
from reading.

Once they finish college, they will face sixty years in which virtually all of their real learning✶✶ will come from *what they read*. It is not a favor to the students for you to repeat what the book said. If you do that, it is a disincentive for them to learn to benefit from their reading.

As strange as it sounds, you don't need to teach the material. I've done that work for you. Relax. You can best teach by example. You read your books, while they read theirs.

The best way for you to help is to check their progress when they work on **The Bridges**.

✶ If "real learning" for adults is exemplified by what they see on television—on quiz shows or the educational channels—then the thousands of dollars and the thousands of hours they spent going to college were an utter waste.

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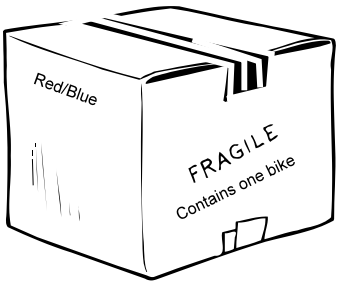
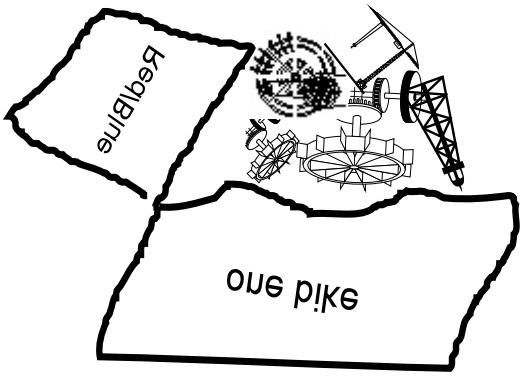
Chapter One

Number Systems

What do five-and-a-half-year-old boys dream about? Many things. For Fred it was a new bicycle. When the box arrived at his office, he tore off the tape. The box fell open. Inside was . . . junk. There were gears, wires, rods, and motors, but no bicycle. He had spent every penny in his checking account ($1,935.06) and didn't get a bike.





Fred had been cheated.

After a short trip with a blanket to a corner of his office to do a little crying, he returned to look at the pile of parts on the floor. There were bags of electrical plugs.

There were springs. Fred thought, What shall I do with all this stuff? Maybe I should just throw it all in the garbage.

Then he almost stepped on a huge remote control. It had about 168 buttons on it. And then it came to him: I know! I will build a robot!

Maybe my $1,935.06 won't be wasted after all.

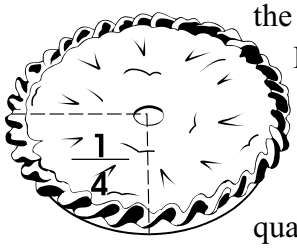
Let's look at $1,935.06 for a moment.

1,935.06

This is a decimal number. That's because it contains a decimal point (the dot between the 5 and the 0).

When you studied the whole numbers, {0, 1, 2, 3, 4, . . . }, you didn't need any decimal points. When you count the number of buttons on a remote control, you get 168, not 168 $\frac{3}{4}$ or 168.75 or –5.

When you cut up a pie into sectors, fractions come in handy. At the dinner table you might ask, "Mom, after you cut Dad's piece—which is one-quarter of the pie—could I please have the rest?"



Your mother, being good in mathematics, does the computation: $1 - \frac{1}{4} = \frac{3}{4}$ and hands you three-quarters of the pie.

But there are times when decimals are more useful than fractions. For example, the bike cost Fred $1,935.06. You could write that as $1935 $\frac{6}{100}$ but that looks a lot messier.

Can you imagine what a car odometer✶ would look like if instead of displaying:

40528.0
40528.1
40528.2
40528.3

it displayed fractions like:

40528 $\frac{1}{8}$
40528 $\frac{1}{4}$
40528 $\frac{1}{3}$
40528 $\frac{1}{2}$
40528 $\frac{9}{16}$

✶ An odometer is the gauge that tells you how far you have gone.

In our number system the position of the digits makes a difference. Would you rather have $18 or $81? Both have the numerals *1* and *8*, but where the *1* and *8* sit makes a big difference.✶

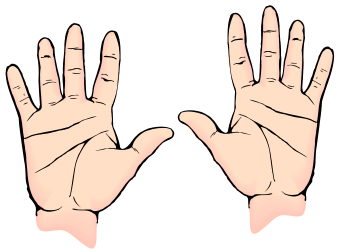
We use the **base ten** system. When we look at a number like

1,935.06

the digit to the left of the decimal (the *5* in this case) is 5 ones. As we move to the left, each digit is "worth" ten times as much. As we move to the right, each digit is "worth" one-tenth as much.

1,935.06 = 1 thousand
+ 9 hundred
+ 3 tens
+ 5 ones
+ 0 tenths
+ 6 hundredths.

We could say that the base ten system is "handy" because—well, look for yourself:



But other base systems have been used over the years. Many ancient cultures used a base 20 system✶✶ (fingers and toes). In the vigesimal system, when you wrote **35**, that meant 3 score + 5 ones. A **score** means 20. So **35** in the base 20 system is the same as 65 in the base ten system.

Traces of the vigesimal system remain in President Lincoln's famous words, "Four score and seven years ago. . . ."

It's time to take a little break. It's time for *Your Turn to Play*. I've been having all the fun so far. It's only fair that you get your chance.

The answers are listed right after all the questions, but please play with the questions a little bit (that is, answer them in writing) before you look at the answers.

✶ In fancy language, we call this a **place-value system** or, even fancier, a **positional numeration system**.

✶✶ More fancy language: vigesimal numeration system [vy JESH eh mul].

Your Turn to Play

1. Write 87 in the vigesimal system.

2. Another really popular numeration system was the base 12 (duodecimal system). There are lots of places in everyday life that reflect the old base 12 system. Can you name three?

3. The oldest known place value system is the Babylonian sexagesimal system (base 60). Can you think of a couple of places in everyday life today that reflect that old system?

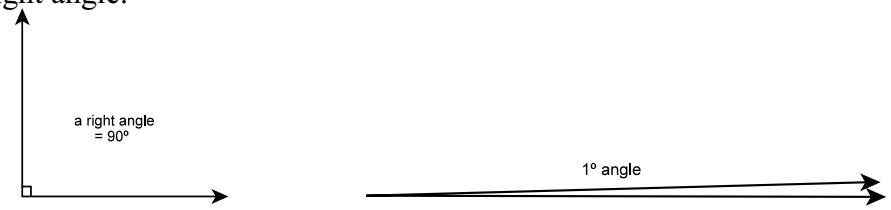
4. $4\frac{2}{3} - 2\frac{3}{4} = ?$

Answers

1. 87 = 4 score + 7 = **47**.

2. Look at a clock (12 hours). Look at a calendar (12 months). Look at a ruler (12 inches). Look at a jury (12 people). Look at eggs (dozen). Look at gold (12 troy ounces = 1 pound).

3. Did you ever wonder why there are sixty seconds in a minute, and sixty minutes in an hour? Now you know. In geometry we will study angles. A one-degree angle (written 1º) is very small. It takes 90 of them to make a right angle.



If you get out your microscope, and picture splitting a one-degree angle into 60 angles, each of those is called a minute. Sixty minutes of angle equals one degree. If you take an angle that measures one minute and split it into 60 little angles, each of those would measure one second. These are super tiny angles.

4. $4\frac{2}{3} - 2\frac{3}{4} = 4\frac{8}{12} - 2\frac{9}{12} = 3\frac{12}{12} + \frac{8}{12} - 2\frac{9}{12} = 1\frac{11}{12}$

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